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Dark matter constraints in heterotic M-theory with five-brane dominance

David Bailin and George V. Kraniotis

Centre for Theoretical Physics, University of Sussex
Brighton BN1 9QJ, U.K.
E-mail: D.Bailin@sussex.ac.uk, G.Kraniotis@sussex.ac.uk

Alex Love

Centre for Particle Physics, Royal Holloway and Bedford New College
University of London, Egham
Surrey TW20-0EX, U.K.
E-mail: A.Love@rhul.ac.uk

Abstract: The phenomenological implications of the M-theory limit in which supersymmetry is broken by the auxiliary fields of five-brane moduli is investigated. Assuming that the lightest neutralino provides the dark matter in the universe, constraints on the sparticle spectrum are obtained. Direct detection rates for dark matter are estimated.

Keywords: Dark Matter, M-Theory, Supersymmetric Standard Model.

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One of the most interesting developments in M-theory [1]–[10] model building is that new non-perturbative tools have been developed which allow the construction of realistic three generation models [11]. In particular the inclusion of five-brane moduli Z_n, (which do not have a weakly coupled string theory counterpart) besides the metric moduli T, S in the effective action leads to new types of $E_8 \times E_8$ symmetry breaking patterns as well as to novel gauge and Kähler threshold corrections. As a result the soft-supersymmetry breaking terms differ substantially from the weakly coupled string.

The phenomenological implications of the effective action of M-theory with the standard embedding of the spin connection into the gauge fields have been investigated in [12]–[16]. Some phenomenological implications of non-standard embeddings in M-theory with and without five-branes have been studied in [17, 18].

In a previous letter, we investigated the supersymmetric particle spectrum in the interesting case when the auxiliary fields associated with the five-branes dominate those associated with metric moduli ($F^{Z_n} \gg F^S, F^T$), including the constraint of radiative electroweak symmetry-breaking [19]. It is the purpose of this paper, to extend the calculation and take into account cosmological constraints on the relic abundance of the neutralino assuming it provides the dark matter of the universe in the region of the parameter space in which it is the lightest supersymmetric particle (LSP). As we shall see in what follows these constraints are quite restrictive.

The soft supersymmetry-breaking terms are determined by the following functions of the effective supergravity theory [11, 18]:

$$
\begin{aligned}
K &= -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) + K_5 + \frac{3}{T+\bar{T}}\left(1+\frac{1}{3}e_O\right) H_{pq} C^p_O \bar{C}^q_O + \\
&\quad + \frac{3}{T+\bar{T}}\left(1+\frac{1}{3}e_H\right) Z_{rs} C^r_H \bar{C}^s_H\,, \\
f_O &= S + B_O T\,, \qquad f_H = S + B_H T\,, \\
W_O &= d_{pqr} C^p_O C^q_O C^r_O\,,
\end{aligned} \tag{1}
$$

where K is the Kähler potential, W_O the observable sector perturbative superpotential, $C^p_O (C^r_H)$ are observable (hidden) sector matter fields and f_O, f_H are the gauge kinetic functions for the observable and hidden sector gauge groups respectively. K_5 is the Kähler potential for the five-brane moduli Z_n and H_{pq} and Z_{rs} are T-independent metrics. $e_{O,H}$ parametrize the relative volumes $V_{O,H}$ of the Calabi-Yau spaces associated respectively with the observable and hidden sectors at the orbifold fixed point. They are given by,

$$
e_O = b_O \frac{T+\bar{T}}{S+\bar{S}}\,, \qquad e_H = b_H \frac{T+\bar{T}}{S+\bar{S}} \tag{2}
$$

and the coefficients $b_{O,H}, B_{O,H}$ are given in terms of the instanton numbers $\beta_{O,H}$ and

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the five brane charges β_n by the following expressions

$$\begin{aligned} b_O &= \beta_O + \sum_{n=1}^{N}(1-z_n)^2\beta_n \\ B_O &= \beta_O + \sum_{n=1}^{N}(1-Z_n)^2\beta_n \\ B_H &= \beta_H + \sum_{n=1}^{N}(Z_n)^2\beta_n \\ b_H &= \beta_H + \sum_{n=1}^{N} z_n^2\beta_n \end{aligned} \tag{3}$$

and the five-brane moduli are denoted by Z_n where $\mathrm{Re}\, Z_n \equiv z_n = x_n/\pi\rho \in (0,1)$ are the five-brane positions in the normalized orbifold coordinates. Since a Calabi-Yau manifold is compact, the net magnetic charge due to orbifold planes and 5-branes is zero. Consequently the following cohomology condition is satisfied

$$\beta_O + \sum_{n=1}^{N}\beta_n + \beta_H = 0 \tag{4}$$

S, T are the dilaton and Calabi-Yau moduli fields and C^p charged matter fields. The superpotential and the gauge kinetic functions are exact up to non-perturbative effects. The standard embedding requires that $\beta_O = -\beta_H$ is positive. The presence of five-branes necessarily entails a non-standard embedding, and we shall be particularly interested in cases where $\beta_O < 0$, which corresponds to an observable sector coupling that is stronger than that of the hidden sector.

Given eqs. (1) one can determine [11, 18] the soft supersymmetry breaking terms for the observable sector gaugino masses $M_{1/2}$, scalar masses m_0 and trilinear scalar couplings A as functions of the auxiliary fields F^S,F^T, F^n of the moduli S, T fields and five-brane moduli Z_n respectively.

$$\begin{aligned} M_{1/2} &= \frac{1}{(S+\bar{S})(1+\frac{B_O T+\bar{B}_O\bar{T}}{S+\bar{S}})}(F^S + F^T B_O + T F^n \partial_n B_O) \\ m_0^2 &= V_0 + m_{3/2}^2 - \frac{1}{(3+e_O)^2}\left[e_O(6+e_O)\frac{|F^S|^2}{(S+\bar{S})^2} + 3(3+2e_O)\frac{|F^T|^2}{(T+\bar{T})^2} - \right. \\ &\qquad - \frac{6e_O}{(S+\bar{S})(T+\bar{T})}\,\mathrm{Re}\, F^S \bar{F}^{\bar{T}} + \\ &\qquad + \left(\frac{e_O}{b_O}(3+e_O)\partial_n\partial_{\bar{m}} b_O - \frac{e_O^2}{b_O^2}\partial_n b_O \partial_{\bar{m}} b_O\right) F^n \bar{F}^{\bar{m}} - \\ &\qquad \left. - \frac{6e_O}{b_O}\frac{\partial_{\bar{n}} b_O}{S+\bar{S}}\,\mathrm{Re}\, F^S \bar{F}^{\bar{n}} + \frac{6e_O}{b_O}\frac{\partial_{\bar{n}} b_O}{T+\bar{T}}\,\mathrm{Re}\, F^T \bar{F}^{\bar{n}} \right], \\ A &= -\frac{1}{3+e_O}\left\{ \frac{F^S(3-2e_O)}{S+\bar{S}} + \frac{3e_O F^T}{T+\bar{T}} + F^n\left(3\frac{e_O}{b_O}\partial_n b_O - (3+e_O)\partial_n K_5\right)\right\}, \end{aligned} \tag{5}$$

where $\partial_n \equiv \partial/\partial Z_n$. The bilinear B-parameter associated with a non-perturbatively generated μ term in the superpotential is given by [18]:

$$B_\mu = \frac{F^S(e_O-3)}{(3+e_O)(S+\bar{S})} - \frac{3(e_O+1)F^T}{(T+\bar{T})(3+e_O)} + \\ + \frac{1}{3+e_O}\left[(3+e_O)F^n\partial_n K_5 - 2F^n\frac{e_O}{b_O}\partial_n b_O\right] - m_{3/2} \qquad (6)$$

From now on we assume that only one five-brane contributes to supersymmetry-breaking.[1] Then the auxiliary fields are given by [18, 22]

$$\begin{aligned} F^1 &= \sqrt{3}m_{3/2}C(\partial_1\partial_{\bar{1}}K_5)^{-1/2}\sin\theta_1 \\ F^S &= \sqrt{3}m_{3/2}C(S+\bar{S})\sin\theta\cos\theta_1 \\ F^T &= m_{3/2}C(T+\bar{T})\cos\theta\cos\theta_1\,. \end{aligned} \qquad (7)$$

The goldstino angles are denoted by θ, θ_1, $m_{3/2}$ is the gravitino mass and $C^2 = 1 + V_0/3m_{3/2}^2$ with V_0 the tree level vacuum energy density. The five-brane dominated supersymmetry-breaking scenario corresponds to $\theta_1 = \pi/2$, i.e. $F^T, F^S = 0$, and we take the five brane which contributes to supersymmetry breaking to be located at $z_1 = 1/2$ in the orbifold interval. We also set $C = 1$ in the above expressions assuming zero cosmological constant. After imposing the requirement of correct electroweak symmetry breaking eq. (6) requires $18 \leq \tan\beta \leq 20.5$ at the electroweak scale (and $-0.7 \leq e_O \leq -0.2$) and also fixes the sign of μ to be negative. However, eq. (6) depends on the mechanism for generation of the μ term in the non-perturbative superpotential, and we prefer to regard B_μ, and therefore $\tan\beta$ and sign μ, as independent parameters in the first instance. However, we note the stronger conclusions that follow when (6) is imposed.

The resulting supersymmetric particle spectrum for a single five-brane present with $z_1 = 1/2$ has been investigated in [19]. Our parameters are, e_O, $\partial_1\partial_{\bar{1}}K_5$, $\partial_1 K_5$, $m_{3/2}$, sign μ (which is not determined by the radiative electroweak symmetry breaking constraint), where μ is the Higgs mixing parameter in the low energy superpotential. The ratio of the two Higgs vacuum expectation values $\tan\beta = \langle H_2^0\rangle/\langle H_1^0\rangle$ is also a free parameter if we leave B to be determined by the minimization of the one-loop Higgs effective potential. If B instead is given by (6), one determines the value of $\tan\beta$. For this purpose we take μ independent of T and S because of our lack of knowledge of μ in M-theory. We treat e_O as a free parameter as the problem of stabilizing the dilaton and other moduli has not yet been solved, although there has been an interesting work in this area [23].

[1]The effects of including more than one 5-brane are difficult to assess because of the extra parameters introduced. Our choice of only one five-brane (in the middle of the orbifold interval) is designed to maximise the effect of including a single five-brane. We also assume negligible CP-violating phases in the soft terms.

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The instanton numbers are model dependent. In this paper we choose to work with the interesting example [18] with $\beta_O = -2$ and $\beta_1 = 1$ which implies $b_O = -7/4$. This implies that $b_H = 5/4$ and allows us to study the region of parameter space with $-1 < e_O \leq 0$, which is not accessible in strongly coupled M-theory scenarios with the standard embedding. We also choose $\partial_1 K_5 = \partial_1 \partial_{\bar{1}} K_5 = 1$. However, we repeated the calculations for $\partial_1 K_5 = 2, 1, 0.5$ and $\partial_1 \partial_{\bar{1}} K_5 = 2, 1, 0.5$, and we comment below on the effect of these deviations.

We use the following experimental bounds from unsuccessful searches at LEP and Tevatron for supersymmetric particles [20]. We require the lightest chargino $M_{\chi_1^+} \geq 90\,\mathrm{GeV}$, and the lightest Higgs, $m_{h_0} \geq 91\,\mathrm{GeV}$ [27]. A lower limit on the mass of the lightest stop $m_{\tilde{t}_2} > 86\,\mathrm{GeV}$, from $\tilde{t}_2 \to c\chi_1^0$ decay in D0 is imposed. The stau mass eigenstate ($\tilde{\tau}$) should be heavier than 81 GeV from LEP2 results.

The soft masses start running from a mass $R_{11}^{-1} \sim 7.5 \times 10^{15}\,\mathrm{GeV}$ with R_{11} the extra M-theory dimension. Then using (5), (6) as boundary conditions for the soft terms, one evolves the renormalization group equations down to the weak scale and determines the sparticle spectrum compatible with the constraints of correct electroweak symmetry breaking and the above experimental constraints on the sparticle spectrum.

Electroweak symmetry breaking is characterized by the extrema equations

$$\begin{aligned} \frac{1}{2}M_Z^2 &= \frac{\bar{m}_{H_1}^2 - \bar{m}_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 \\ -B\mu &= \frac{1}{2}(\bar{m}_{H_1}^2 + \bar{m}_{H_2}^2 + 2\mu^2)\sin 2\beta \end{aligned} \tag{8}$$

where

$$\bar{m}_{H_1,H_2}^2 \equiv m_{H_1,H_2}^2 + \frac{\partial \Delta V}{\partial v_{1,2}^2} \tag{9}$$

and $\Delta V = (64\pi^2)^{-1}\,\mathrm{STr}\,M^4[ln(M^2/Q^2) - 3/2]$ is the one loop contribution to the Higgs effective potential. We include contributions only from the third generation of particles and sparticles.

Since $\mu^2 \gg M_Z^2$ for most of the allowed region of the parameter space [24], the following approximate relationships hold at the electroweak scale for the masses of neutralinos and charginos, which of course depend on the details of electroweak symmetry breaking.

$$\begin{aligned} m_{\chi_1^\pm} &\sim m_{\chi_2^0} \sim 2m_{\chi_1^0} \\ m_{\chi_{3,4}^0} &\sim m_{\chi_2^\pm} \sim |\mu| \end{aligned} \tag{10}$$

In (10) $m_{\chi_{1,2}^\pm}$ are the chargino mass eigenstates and $m_{\chi_i^0}$, $i = 1, \ldots, 4$ are the four neutralino mass eigenstates with $i = 1$ denoting the lightest neutralino. The former arise after diagonalization of the mass matrix.

$$M_{ch} = \begin{pmatrix} M_2 & \sqrt{2}m_W \sin\beta \\ m_W \cos\beta & -\mu \end{pmatrix} \tag{11}$$

where M_2 denotes the weak gaugino mass and M_1 will denote the $\mathrm{U}(1)_Y$ gaugino (Bino) mass. The stau mass matrix is given by the expression

$$\mathcal{M}_{\tau}^2 = \begin{pmatrix} \mathcal{M}_{11}^2 & m_{\tilde{\tau}}(A_\tau + \mu \tan\beta) \\ m_{\tilde{\tau}}(A_\tau + \mu \tan\beta) & \mathcal{M}_{22}^2 \end{pmatrix} \tag{12}$$

where $\mathcal{M}_{11} = m_L^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2)\cos 2\beta$ and $\mathcal{M}_{22} = m_E^2 + m_\tau^2 + (M_W^2 - M_Z^2)\cos 2\beta$, and m_L^2, m_E^2 refer to scalar soft masses for lepton doublet, singlet respectively.

As has been first noted in [18], in the case when only the five-branes contribute to supersymmetry-breaking the ratio of scalar masses to gaugino mass, $m_0/|M_{1/2}| > 1$ for $e_O > -0.65$. This is quite interesting since scalar masses larger than gaugino masses are not easy to obtain in the weakly-coupled heterotic string or M-theory compactification with standard embedding. As we shall see cosmological constraints become important in this region of the parameter space. In the case of non-standard embeddings without five-branes there is small region of the parameter space where is possible to have $m_0/|M_{1/2}| > 1$.

Assuming R-parity conservation the LSP is stable, and consequently if it is neutral can provide a good dark matter candidate. We assume that the dark matter is in the form of neutralinos. The lightest neutralino is a linear combination of the superpartners of the photon, Z^0 and neutral-Higgs bosons,

$$\chi_1^0 = N_{11}\tilde{B} + N_{12}\tilde{W}^3 + N_{13}\tilde{H}_1^0 + N_{14}\tilde{H}_2^0 \tag{13}$$

The neutralino 4×4 mass matrix can be written as

$$\begin{pmatrix} M_1 & 0 & -M_Z A_{11} & M_Z A_{21} \\ 0 & M_2 & M_Z A_{12} & -M_Z A_{22} \\ -M_Z A_{11} & M_Z A_{12} & 0 & \mu \\ M_Z A_{21} & -M_Z A_{22} & \mu & 0 \end{pmatrix}$$

with

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} \sin\theta_W \cos\beta & \cos\theta_W \cos\beta \\ \sin\theta_W \sin\beta & \cos\theta_W \sin\beta \end{pmatrix}$$

When the observational data on temperature fluctuations, type-Ia supernovae, and gravitational lensing are combined with popular cosmological models, the dark matter relic abundance (Ω_{LSP}) typically satisfies [28]

$$0.1 \le \Omega_{\mathrm{LSP}} h^2 \le 0.4\,, \tag{14}$$

where h is the reduced Hubble constant.

We calculated the relic abundance of the lightest neutralino in the scenarios we have considered using standard techniques [25]. When these results are confronted with the (model-dependent) bounds (14) derived from the observational data further constraints on the parameters $m_{3/2}, \tan\beta, \mu, e_O$ are obtained and these give new constraints on the sparticle spectrum.

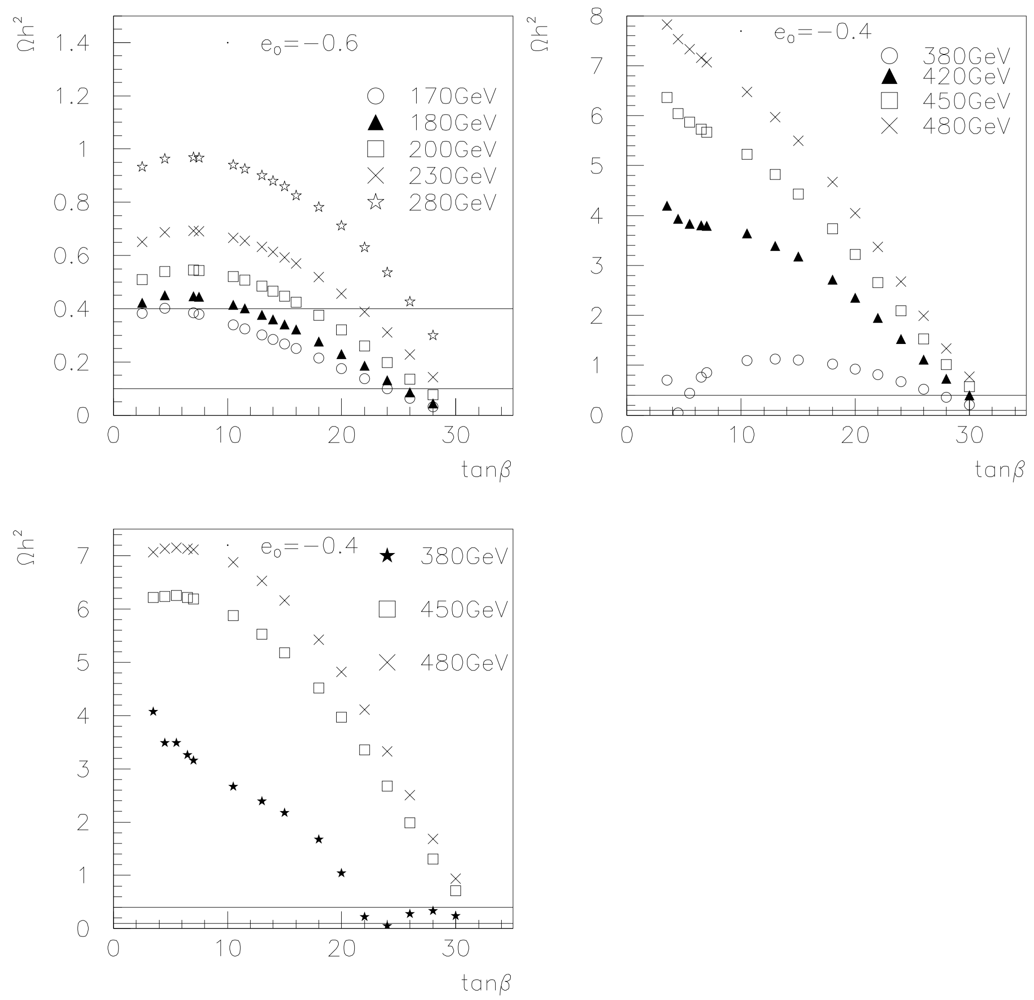

Figure 1: Relic abundance of the LSP versus $\tan\beta$ for various values of the gravitino mass and $e_O = -0.6, \mu < 0, e_O = -0.4, \mu < 0, \mu > 0$. We also exhibit the upper and lower cosmological bounds on the LSP relic abundance.

In figure 1 we display the relic abundance of the lightest neutralino versus $\tan\beta$ for different values of the gravitino mass and the parameter e_O. We see that $\Omega_{\rm LSP}h^2 \leq 0.4$ puts $m_{3/2}$ dependent lower bounds on the values of $\tan\beta$. Let us start the discussion with the case $e_0 = -0.6, \mu < 0$, first graph of figure 1. In this case the upper limit on the relic abundance provides lower bounds on $\tan\beta$. For instance, for $m_{3/2} = 170\,\mathrm{GeV}$ $\tan\beta > 5$ while for $m_{3/2} = 230\,\mathrm{GeV}$ $\tan\beta > 20$. Values of the gravitino mass $m_{3/2} < 170\,\mathrm{GeV}$ come into contradiction with the lower experimental bounds on the lightest Higgs mass and lightest chargino mass imposed from unsuccessful searches for supersymmetric particles in accelerator experiments. The lower limit on the relic abundance ($\Omega_{\rm LSP}h^2 > 0.1$) imposes further constraints on the gravitino mass for $\tan\beta \geq 26$. In particularfor $\tan\beta = 26$, $m_{3/2} \geq 182\,\mathrm{GeV}$ which results in: $m_{\chi_1^+} \geq 102\,\mathrm{GeV}, m_{h^0} \geq 115\,\mathrm{GeV}$. Similarly for $\tan\beta = 28$, $m_{3/2} \geq 210\,\mathrm{GeV}$ and $m_{\chi_1^+} \geq 119.5\,\mathrm{GeV}$. In this case the lightest Higgs mass $m_{h^0} \geq 117.6\,\mathrm{GeV}$. The allowed

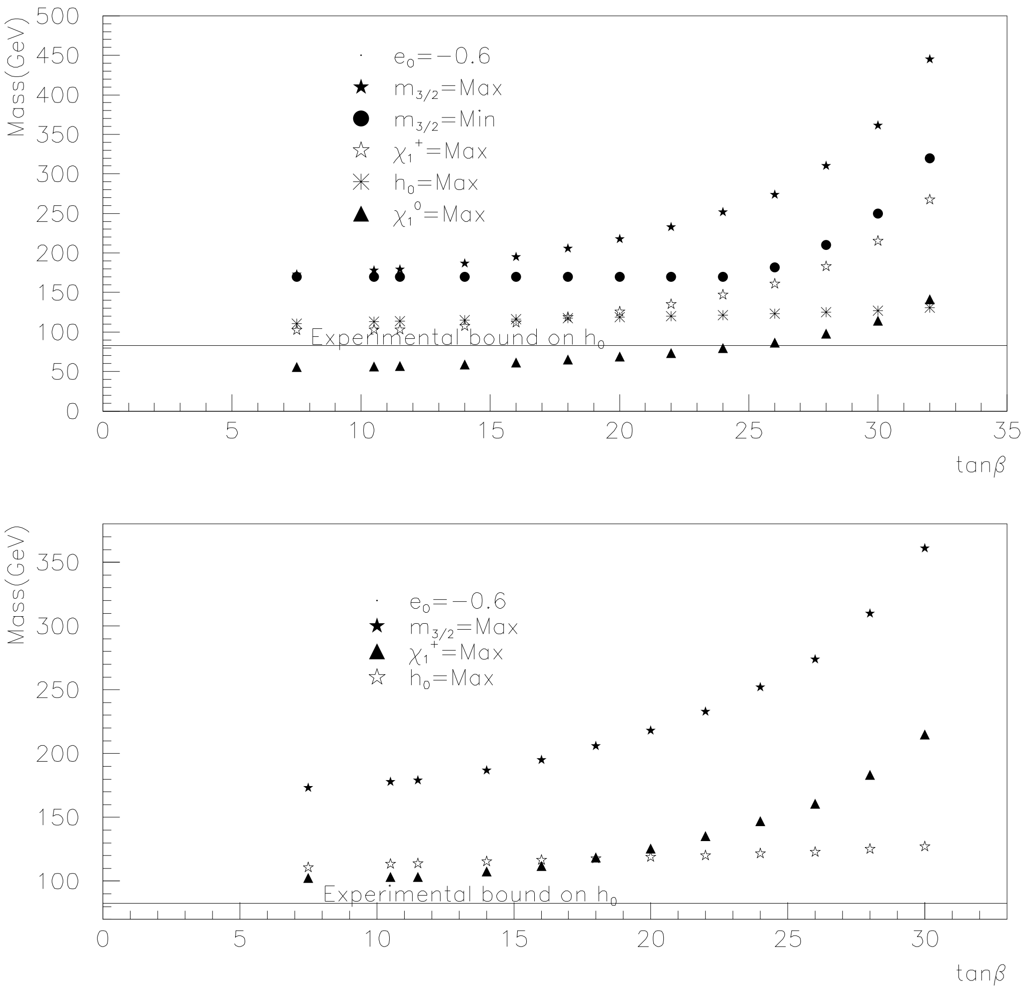

Figure 2: Maximum gravitino, lightest chargino and lightest Higgs masses (imposed by $\Omega h^2 \leq 0.4$) versus $\tan\beta$ for $e_O = -0.6$, $\mu < 0$, $\mu > 0$.

values of $m_{3/2}$ as a function of $\tan\beta$, compatible with (14), and for $e_O = -0.6, \mu < 0$ are shown in figure 2. Also shown in figures 2 are upper bounds on the lightest chargino, lightest Higgs and lightest neutralino masses respectively, compatible with the upper cosmological limit on the relic abundance for both signs of μ. For the calculation of the masses of the CP-even Higgs bosons, h^0 and H, and the CP-odd mass eigenstate A^0, we used one-loop corrected expressions including contributions from the third generation quarks and squarks [26]. We have also checked that the value of the calculated CP-odd Higgs mass is consistent with current experimental bounds [27]. In particular for $e_O = -0.6$, $m_{3/2} = 170\,\text{GeV}$, $\mu < 0$, and for $7.5 \leq \tan\beta \leq 30$, we obtain $146\,\text{GeV} \leq m_{A^0} \leq 305\,\text{GeV}$ (the value of the CP-odd Higgs mass decreases when $\tan\beta$ increases). Current experimental bounds for a typical choice of parameters for the MSSM imply that for $m_{A^0} \geq 150\,\text{GeV}$ values of $\tan\beta > 5$ are allowed [27]. The resulting model can be tested at accelerator experiments.

$\tan\beta$	$m_{3/2}^{\max}$	m_{h^0}	$m^+_{\chi_1}$	$m_{\chi_1^0}$	$m_{\tilde{t}_2}$	$m_{\tilde{\tau}_2}$
22	380 GeV	118.5 GeV	99 GeV	54 GeV	155 GeV	310 GeV
28	381 GeV	118 GeV	99 GeV	54 GeV	154 GeV	284 GeV
30	400 GeV	119 GeV	104 GeV	56 GeV	164 GeV	288 GeV
32	495 GeV	122 GeV	131 GeV	70 GeV	216 GeV	347 GeV
34	750 GeV	129 GeV	201 GeV	106 GeV	371 GeV	513 GeV

Table 1: Upper bounds on sparticle masses resulting from eq. (14) for $e_O = -0.4, \mu < 0$ for various values of $\tan\beta$.

The cosmological constraints become even more important when $e_O \to 0$. In the last two plots of figure 1 we plot the relic abundance of the lightest neutralino versus $\tan\beta$ for different values of the gravitino mass for $e_O = -0.4$. The lower experimental bounds (from unsuccessful searches in accelerator experiments) on the lightest chargino mass and on the lightest Higgs mass now require that $m_{3/2} \geq 380\,\mathrm{GeV}$. For $\mu < 0$ we see that $\Omega_{\mathrm{LSP}}h^2 \leq 0.4$ requires $\tan\beta \geq 22$ for any allowed value of the gravitino mass and for $22 \leq \tan\beta \leq 30$, the relic abundance of the lightest neutralino is in the range: $0.05 \leq \Omega_{\mathrm{LSP}}h^2 \leq 0.33$. The upper bounds on the gravitino, lightest chargino, lightest Higgs, lightest stop and lightest stau masses as a function of $\tan\beta$ compatible with the upper limit on the relic abundance are summarised in table 1.

In figure 3 we plot the relic abundance of the lightest neutralino versus e_O for $\tan\beta = 15, 26$ respectively and fixed Bino mass at the unification scale $M_1(M_U) = 126\,\mathrm{GeV}$. This choice corresponds to a lightest neutralino mass of $\sim 54\,\mathrm{GeV}$ corresponding to a lightest chargino mass of about $100\,\mathrm{GeV}$, the current experimental lower bound. From figure 3 we observe that for $e_O \geq -0.55$, $\tan\beta = 15$, $\Omega_{\mathrm{LSP}}h^2 > 0.4$ and for $e_O > -0.5$ is greater than 1. On the other hand for $e_O \to -1$ the cosmological constraints are more easily satisfied although for e_O too close to -1 $\Omega_{\mathrm{LSP}}h^2 > 0.1$ becomes more difficult to satisfy. Also for $e_O < -0.8$ the lightest stau becomes the lightest supersymmetric particle. For $\tan\beta = 26$, we have that for $e_O \geq -0.45$, $\Omega_{\mathrm{LSP}}h^2 > 0.4$.

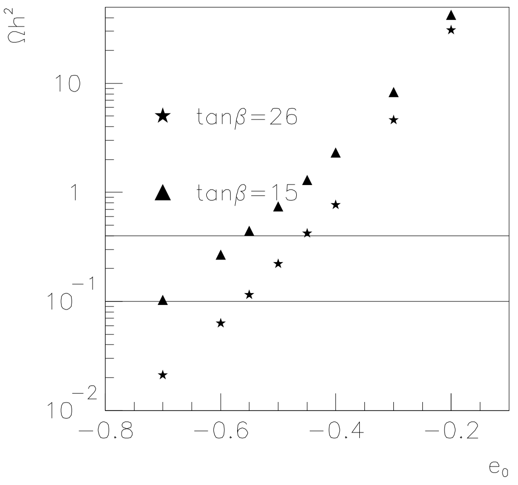

Figure 3: Relic abundance of LSP vs e_O for $\tan\beta = 15, 26, \partial_1 K_5 = \partial_1\partial_{\bar{1}} K_5 = 1, |M_1(M_U)| = 126\,\mathrm{GeV}, \mu < 0.$.

If we impose (the model-dependent) eq. (6) for B_μ, we obtain limits $18 \leq \tan\beta \leq 20.5$ on $\tan\beta$. Then figures 1, 2, 3 show that the cosmological bounds (14) on the dark matter relic abundance are comfortably satisfied when $e_O \leq -0.5$. For $e_O = -0.6$ for example, the gravitino mass is bounded by $170\,\mathrm{GeV} \leq m_{3/2} \leq 200\,\mathrm{GeV}$.

Assuming that the neutralinos provide the cold dark matter in our Galaxy we calculated its direct detection rates for various nuclei. For an LSP moving with velocity v_z with respect to the detector nuclei the detection rate for a target with mass m is given by [25]

$$R = \frac{\rho^{0.3}_{\chi^0_1}}{m_{\chi^0_1}} \frac{m}{A m_p} \int f(\mathbf{v}) |\mathbf{v_z}| \sigma(|\mathbf{v}|) \mathbf{d^3v}\,, \tag{15}$$

where $\rho^{0.3}_{\chi^0_1}$ denotes the local LSP mass density normalized to the standard value of $0.3\,\mathrm{GeVcm}^{-3}$, $f(\mathbf{v})$ a Maxwell velocity distribution and σ denotes the neutralino-nucleus elastic cross section.

For $e_0 = -0.6$ the cross sections of neutralinos with nucleon result in large total detection rates in the cosmologically interesting region for large values of $\tan\beta$. In particular, for $^{73}Ge, ^{208}Pb, ^{131}Xe$ detectors, detection rates of the neutralinos are in the range of order $10^{-3} - O(1)$ events/Kg/day for $\mu < 0$. The larger total event rates occur for $\tan\beta \geq 24$. The results for $\mu > 0$ are similar. This illustrates the fact that $\Omega_{\mathrm{LSP}} h^2 \sim \frac{10^{-37}\,\mathrm{cm}^2}{\langle \sigma_{\mathrm{anni}} v \rangle}$ and the neutralino annihilation cross section is roughly proportional to the neutralino scattering cross section. Thus as the LSP abundance decreases, its scattering cross section generally increases. For $\Omega_{\mathrm{LSP}} h^2 \sim 0.1$ this results in an increased event rate. For values of $e_O > -0.6$ the total detection rates are smaller. This behaviour of the detection rates can be understood by investigating the neutralino-nucleon scalar (spin-independent) cross section, which in this model is the dominant contribution to the total neutralino-nucleus elastic cross section.

The scalar nucleon-LSP cross section is given by [12, 31]

$$\sigma^{(\mathrm{nucleon})}_{\mathrm{scalar}} = \frac{8G_F^2}{\pi} M_W^2 m_{red}^2 \left[\frac{G_1(h_0) I_{h_0}}{m^2_{h_0}} + \frac{G_2(H) I_H}{m^2_H} + \cdots \right]^2 , \tag{16}$$

where

$$\begin{aligned} G_1(h_0) &= (-N_{11}\tan\theta_W + N_{21})(N_{31}\sin\alpha + N_{41}\cos\alpha) \\ G_2(H) &= (-N_{11}\tan\theta_W + N_{21})(-N_{31}\cos\alpha + N_{41}\sin\alpha) \end{aligned} \tag{17}$$

and

$$I_{h_0,H} = \sum_q l_q^{h_0,H} m_q \langle N | \bar{q} q | N \rangle \tag{18}$$

and

$$\begin{aligned} l_q^{h_0} &= \frac{\cos\alpha}{\sin\beta} \qquad & l_q^H &= \frac{\sin\alpha}{\sin\beta} \qquad & &\text{for } q = u, c, t \\ l_q^{h_0} &= -\frac{\sin\alpha}{\cos\beta} \qquad & l_q^H &= \frac{\cos\alpha}{\cos\beta} \qquad & &\text{for } q = d, s, b\,, \end{aligned} \tag{19}$$

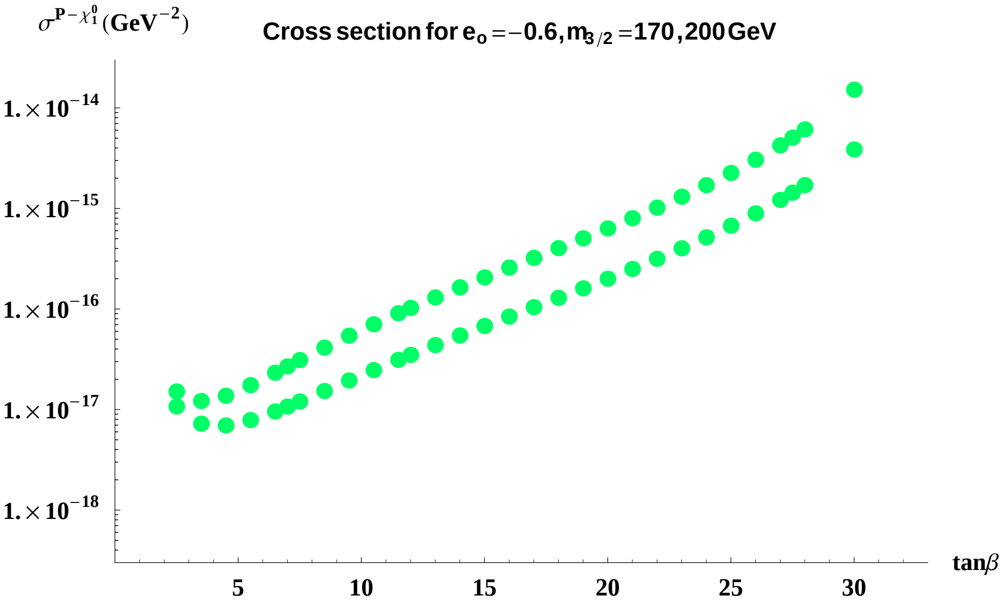

Figure 4: Proton-scalar LSP cross section versus $\tan\beta$ for fixed $e_o = -0.6$, $\mu < 0$, $m_{3/2} = 170$ (upper curve), 200 GeV (lower curve).

where the two first terms inside the brackets refer to the diagrams with h_0 and H-exchanges in the t-channel and the the ellipsis refers to the graphs with squark-exchanges in the s- and u-channels [12]. In equation (16) m_{red} is the neutralino-nucleon reduced mass, h_0, H denote the lightest Higgs and CP-even heavier Higgs respectively and α is the Higgs mixing angle.[2]

Of particular interest is the $\tan\beta$ dependence of the scalar neutralino-nucleon cross section $\sigma^{\rm nucleon}_{\rm scalar}$. For high values of $\tan\beta$ the corresponding cross section generically increases (see figures 4, 5). The calculated cross section for high $\tan\beta$ reaches the sensitivity of current dark matter experiments for the nucleon-neutralino scalar cross section in the range $1 \times 10^{-9}\,\text{nb}\,(2.6{\times}10^{-15}\,\text{GeV}^{-2}) \leq \sigma^{\rm (nucleon)}_{\rm scalar} \leq 3{\times}10^{-8}\,\text{nb}$ $(7.7 \times 10^{-14}\,\text{GeV}^{-2})$ [31]. The larger the value of the e_O parameter the smaller the cross-section for fixed $\tan\beta$, sign μ and fixed bino mass. Also the cross section decreases for increasing values of the gravitino mass for fixed e_O and sign μ as is evident from figures 4, 5. The two upper curves in graphs 4 and 5 are cross-sections for an LSP mass about 54 GeV. The corresponding spin-dependent cross sections [12] are much smaller by two to three orders of magnitude. If we impose (6), the required large value of $\tan\beta$ implies a large neutralino-nucleon scalar cross section, and hence predicts high dark matter detection rates.

We now comment on the effect of deviating from the choice $\partial_1 K_5 = \partial_1\partial_{\bar{1}} K_5 = 1$. With regard to the sparticle spectrum, for fixed value of e_O and fixed value of $\partial_1 K_5$ the effect of increasing the value of $\partial_1\partial_{\bar{1}} K_5$ is that the ratio $|m_0/M_{1/2}|$ increases.

[2]We determine the Higgs mixing angle numerically by diagonalizing the one-loop CP-even Higgs mass matrix.

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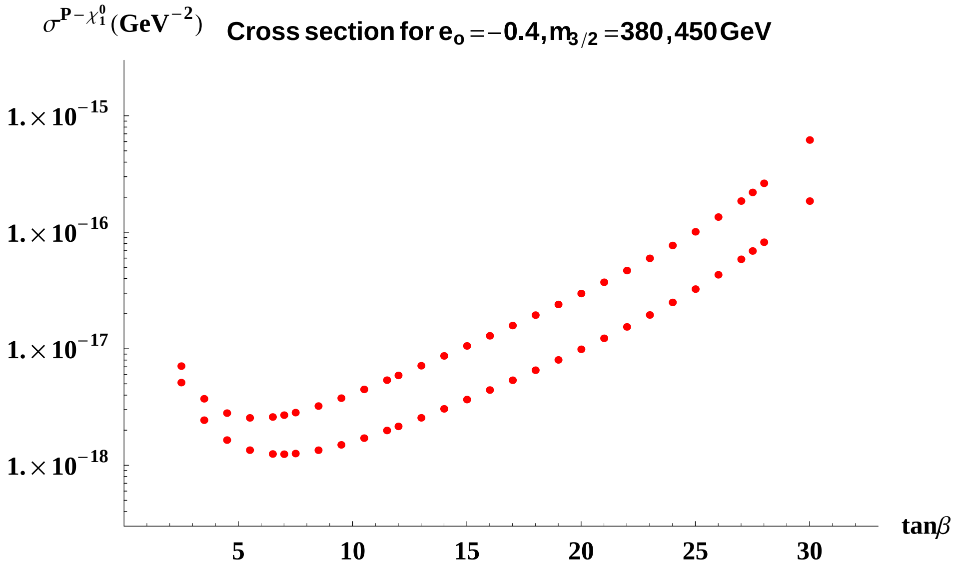

Figure 5: Proton-LSP cross section versus $\tan\beta$ for $e_o = -0.4$, $\mu < 0$, $m_{3/2} = 380\,\mathrm{GeV}$ (upper curve), $450\,\mathrm{GeV}$ (lower curve).

Thus for large values of $\partial_1\partial_{\bar{1}}K_5$, the ratio $|m_0/M_{1/2}| \gg 1$ which is equivalent to the case $e_O \to 0$. This can be understood from the explicit dependence of the scalar masses and gaugino masses on the parameter $\partial_1\partial_{\bar{1}}K_5$. This parameter appears in the denominator in both soft supersymmetry-breaking terms and as its value becomes very large, the gaugino masses tend to 0 while the scalar masses tend to the gravitino mass. For low values of $\partial_1\partial_{\bar{1}}K_5$ the ratio of the scalar masses to gaugino masses decreases and for high $\tan\beta$ the lightest stau can become the LSP, as in the M-theory without five-branes case [12]. This scenario is excluded in this paper as we assume that the LSP is neutral. The effect of varying the first derivative $\partial_1 K_5$ on the sparticle spectrum is the following; This parameter appears only in the trilinear soft supersymmetry-breaking A-terms. For fixed value of $\partial_1\partial_{\bar{1}}K_5$, fixed gravitino mass and fixed $e_O = -0.6$ and in the low $\tan\beta$ region, i.e. $\tan\beta = 7.5$, the lightest chargino is the next to lightest supersymmetric particle (NLSP) with the lightest stau the NNLSP for $\partial_1 K_5 = 0.5, 1$, while for $\partial_1 K_5 = 2$ the lightest stop becomes the NLSP. In the high $\tan\beta$ region for a fixed value of $\partial_1\partial_{\bar{1}}K_5$, fixed gravitino mass and fixed $e_O = -0.6$, the lightest chargino or the lightest stau is the NLSP for $\partial_1 K_5 = 0.5, 1$ while for $\partial_1 K_5 = 2$ the lightest chargino is the NLSP, with the lightest stop the NNSLP. In both cases increasing $\partial_1 K_5$ reduces the mass of the lightest stop eigenstate, and for low $\tan\beta$ it becomes the NLSP while for high $\tan\beta$ it becomes the NNLSP. With regard to dark matter the same qualitative features as were discussed above are still present, namely large dark matter cross-sections are associated with large $\tan\beta$. However, $\partial_1\partial_{\bar{1}}K_5$ is constrained by the bounds (14) on the relic abundance. The increasing value of $|m_0/M_{1/2}|$ as $\partial_1\partial_{\bar{1}}K_5$ increases has the efffect of increasing the relic abundance and for low values of $\tan\beta$ this will lead to an upper bound on $\partial_1\partial_{\bar{1}}K_5$. For example for $e_O = -0.6, \tan\beta = 7.5, m_{3/2} = 170\,\mathrm{GeV}$, $\mu < 0, \partial_1 K_5 = 2$, $\partial_1\partial_{\bar{1}}K_{5\max} = 2.4$. For high values of $\tan\beta$ the relic abundance is typically near the lower bound in eq. (14), unless $\partial_1\partial_{\bar{1}}K_5$ is huge, and for low values of $\partial_1\partial_{\bar{1}}K_5$ either the lower bound is not satisfied or the lightest stau is the LSP. For example for $e_O = -0.6$, $\tan\beta = 30, \mu < 0, m_{3/2} = 140\,\mathrm{GeV}$, $\partial_1 K_5 = \partial_1\partial_{\bar{1}}K_5 = 0.5$ the lightest stau is the LSP while for $e_O = -0.6$, $\tan\beta = 30, \mu < 0, m_{3/2} = 180\,\mathrm{GeV}$, $\partial_1 K_5 = 0.5$ and $\partial_1\partial_{\bar{1}}K_5 = 1$, the lightest neutralino is the LSP and $\Omega_{\mathrm{LSP}}h^2 = 0.023$.

Recently, large detection rates have been obtained in type-I string theories formulated as orientifold compactifications of type-IIB string theory [29, 30]. In particular, in the mirage unification scenario large detection rates have been obtained. This scenario, differs from the ones studied in this paper in the following respects: first in the mirage unification scenario the lightest neutralino has a large Higgsino component while in the 5-brane dominated limit it is almost a Bino. Thus in the first case besides a large scalar cross-section the LSP has also a rather large spin-dependent couplings with the nuclei. Second, in the type-I model, the nucleon-neutralino cross section is large and consequently the detection rates are large when $\tan\beta$ is small, i.e. $\tan\beta \leq 8$. Also in this case the high neutralino-nucleon cross sections correspond to relatively low relic neutralino densities, i.e. $\Omega_{\mathrm{LSP}}h^2 \leq 0.1$ and therefore another form of dark matter might be needed to close the Universe. As we saw in the current model large cross-sections occur in the high $\tan\beta$ region. Thus the two models lead to different predictions.

In conclusion, as well as predictions for direct detection for the lightest neutralino χ_1^0 we have obtained bounds on the value of $\tan\beta$ from the cosmological constraints on the relic abundance. We have also calculated the maximum lightest chargino mass and the lightest maximum Higgs mass as a function of $\tan\beta$ for various values of the ratio of the scalar masses to gaugino masses. The resulting sparticle spectra should be tested in accelerator experiments in Tevatron and LHC. There is some model dependence in our results because we have restricted attention to a model with instanton numbers $\beta_O = -2$ and $\beta_1 = 1$ in order to study the region of the parameter space with $-1 < e_O \leq 0$, which is not accessible in strongly coupled M-theory scenarios with standard embedding, and to the simplest case of a single five-brane. Because of our lack of knowledge of K_5 we have also assumed $\partial_1 K_5 = \partial_1\partial_{\bar{1}}K_5 = 1$. However, we repeated the calculations for $\partial_1 K_5 = 2, 1, 0.5$ and $\partial_1\partial_{\bar{1}}K_5 = 2, 1, 0.5$. For dark matter cross-sections the same qualitative features as were found earlier survive.

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References

[1] P. Hořava and E. Witten, *Heterotic and type-I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209]; *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys.* **B 475** (1996) 94 [hep-th/9603142].

[2] E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070].

[3] I. Antoniadis and M. Quiros, *Large radii and string unification*, *Phys. Lett.* **B 392** (1997) 61 [hep-th/9609209]; *Supersymmetry breaking in M-theory and gaugino condensation*, *Nucl. Phys.* **B 505** (1997) 109 [hep-th/9705037].

[4] T.-J. Li, J.L. Lopez and D.V. Nanopoulos, *M-theory inspired no-scale supergravity*, *Mod. Phys. Lett.* **A 12** (1997) 2647 [hep-ph/9702237];
Compactifications of M-theory and their phenomenological consequences, *Phys. Rev.* **D 56** (1997) 2602 [hep-ph/9704247];
D.V. Nanopoulos, *M-phenomenology*, hep-th/9711080.

[5] T.-J. Li, *Compactification and supersymmetry breaking in M-theory*, *Phys. Rev.* **D 57** (1998) 7539 [hep-th/9801123].

[6] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Supersymmetry breaking and soft terms in M-theory*, *Phys. Lett.* **B 415** (1997) 24 [hep-th/9707143]; *Supersymmetry breakdown at a hidden wall*, *Nucl. Phys.* **B 530** (1998) 43 [hep-th/9801030].

[7] E. Dudas and C. Grojean, *Four-dimensional M-theory and supersymmetry breaking*, *Nucl. Phys.* **B 507** (1997) 553 [hep-th/9704177];
E. Dudas, *Supersymmetry breaking in M-theory and quantization rules*, *Phys. Lett.* **B 416** (1998) 309 [hep-th/9709043].

[8] Z. Lalak and S. Thomas, *Gaugino condensation, moduli potential and supersymmetry breaking in M-theory models*, *Nucl. Phys.* **B 515** (1998) 55 [hep-th/9707223].

[9] K. Choi, H.B. Kim and C. Muñoz, *Four-dimensional effective supergravity and soft terms in M-theory*, *Phys. Rev.* **D 57** (1998) 7521 [hep-th/9711158].

[10] A. Lukas, B.A. Ovrut and D. Waldram, *Gaugino condensation in M-theory on S^1/Z_2*, *Phys. Rev.* **D 57** (1998) 7529 [hep-th/9711197];
A. Lukas, B.A. Ovrut and D. Waldram, *On the four-dimensional effective action of strongly coupled heterotic string theory*, *Nucl. Phys.* **B 532** (1998) 43 [hep-th/9710208].

[11] A. Lukas, B.A. Ovrut and D. Waldram, *Non-standard embedding and five-branes in heterotic M-theory*, *Phys. Rev.* **D 59** (1999) 106005 [hep-th/9808101]; *Five-branes and supersymmetry breaking in M-theory*, *J. High Energy Phys.* **04** (1999) 009 [hep-th/9901017];

R. Donagi, A. Lukas, B.A. Ovrut and D. Waldram, *Holomorphic vector bundles and non-perturbative vacua in M-theory*, *J. High Energy Phys.* **06** (1999) 034 [hep-th/9901009].

[12] D. Bailin, G.V. Kraniotis and A. Love, *Sparticle spectrum and dark matter in M-theory*, *Phys. Lett.* **B 432** (1998) 90 [hep-ph/9803274];
Nucl. Phys. **B 556** (1999) 23.

[13] C.-S. Huang, T.-j. Li, W. Liao, Q.-S. Yan and S.H. Zhu, *Scales, couplings revisited and low energy phenomenology in M-theory on S^1/Z_2*, *Eur. Phys. J.* **C 18** (2000) 393 [hep-ph/9810412].

[14] S.A. Abel and C.A. Savoy, *Charge and colour breaking constraints in the MSSM with non-universal SUSY breaking*, *Phys. Lett.* **B 444** (1998) 119 [hep-ph/9809498].

[15] J.A. Casas, A. Ibarra and C. Muñoz, *Phenomenological viability of string and M-theory scenarios*, *Nucl. Phys.* **B 554** (1999) 67 [hep-ph/9810266].

[16] T. Li, *Phenomenology remarks in M-theory on S^1/Z_2*, hep-ph/9903371.

[17] T. Kobayashi, J. Kubo and H. Shimabukuro, *Electroweak symmetry breaking and S-spectrum in M-theory*, *Nucl. Phys.* **B 580** (2000) 3 [hep-ph/9904201].

[18] D.G. Cerdeño and C. Muñoz, *Phenomenology of non-standard embedding and five-branes in M-theory*, *Phys. Rev.* **D 61** (2000) 016001 [hep-ph/9904444].

[19] D. Bailin, G.V. Kraniotis and A. Love, *Sparticle spectrum in M-theory with five-brane dominance*, *Phys. Lett.* **B 463** (1999) 174 [hep-ph/9906507].

[20] Particle Data Group, *Review of particle physics*, *Eur. Phys. J.* **C 15** (2000) 1.

[21] L.E. Ibáñez and G.G. Ross, $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ *symmetry breaking as a radiative effect of supersymmetry breaking in GUTs*, *Phys. Lett.* **B 110** (1982) 215;
L. Alvarez-Gaume, J. Polchinski and M.B. Wise, *Minimal low-energy supergravity*, *Nucl. Phys.* **B 221** (1983) 495;
J.R. Ellis, J.S. Hagelin, D.V. Nanopoulos and K. Tamvakis, *Weak symmetry breaking by radiative corrections in broken supergravity*, *Phys. Lett.* **B 125** (1983) 275;
M. Claudson, L.J. Hall and I. Hinchliffe, *Low-energy supergravity: false vacua and vacuous predictions*, *Nucl. Phys.* **B 228** (1983) 501;
C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quiros, *Low-energy behavior of realistic locally supersymmetric grand unified theories*, *Nucl. Phys.* **B 236** (1984) 438;
L.E. Ibáñez and C. Lopez, *N=1 supergravity, the weak scale and the low-energy particle spectrum*, *Nucl. Phys.* **B 233** (1984) 511;
A. Bouquet, J. Kaplan and C.A. Savoy, *Low-energy constraints on supergravity parameters*, *Nucl. Phys.* **B 262** (1985) 299.

[22] A. Brignole, L.E. Ibáñez and C. Muñoz, *Towards a theory of soft terms for the supersymmetric standard model*, *Nucl. Phys.* **B 422** (1994) 125 [hep-ph/9308271].

[23] K. Choi, H.B. Kim and H.D. Kim, *Moduli stabilization in heterotic M-theory*, *Mod. Phys. Lett.* **A 14** (1999) 125 [hep-th/9808122].

[24] P. Nath and R. Arnowitt, *Top quark mass and Higgs mass limits in the standard* SU(5) *supergravity unification*, *Phys. Lett.* **B 289** (1992) 368.

[25] G. Jungman, M. Kamionkowski and K. Griest, *Supersymmetric dark matter*, *Phys. Rep.* **267** (1996) 195 [hep-ph/9506380].

[26] J.R. Ellis, G. Ridolfi and F. Zwirner, *Radiative corrections to the masses of supersymmetric Higgs bosons*, *Phys. Lett.* **B 257** (1991) 83;
J.R. Ellis, G. Ridolfi and F. Zwirner, *On radiative corrections to supersymmetric Higgs boson masses and their implications for LEP searches*, *Phys. Lett.* **B 262** (1991) 477.

[27] http://lephiggs.web.cern.ch/LEPHIGGS/papers/index.html.

[28] N. Bahcall, J.P. Ostriker, S. Perlmutter and P.J. Steinhardt, *The cosmic triangle: revealing the state of the universe*, *Science* **284** (1999) 1481

[29] E. Gabrielli, S. Khalil, C. Muñoz and E. Torrente-Lujan, *Initial scales, supersymmetric dark matter and variations of neutralino nucleon cross sections*, *Phys. Rev.* **D 63** (2001) 025008 [hep-ph/0006266].

[30] D. Bailin, G.V. Kraniotis and A. Love, *Sparticle spectrum and dark matter in type-I string theory with an intermediate scale*, *Phys. Lett.* **B 491** (2000) 161 [hep-ph/0007206].

[31] A. Bottino, F. Donato, N. Fornengo and S. Scopel, *Implications for relic neutralinos of the theoretical uncertainties in the neutralino nucleon cross-section*, *Astropart. Phys.* **13** (2000) 215 [hep-ph/9909228];
J.R. Ellis, A. Ferstl and K.A. Olive, *Exploration of elastic scattering rates for supersymmetric dark matter*, *Phys. Rev.* **D 63** (2001) 065016 [hep-ph/0007113];
E. Accomando, R. Arnowitt, B. Dutta and Y. Santoso, *Neutralino proton cross sections in supergravity models*, *Nucl. Phys.* **B 585** (2000) 124 [hep-ph/0001019];
E. Gabrielli, S. Khalil, C. Muñoz and E. Torrente-Lujan, *Initial scales, supersymmetric dark matter and variations of neutralino nucleon cross sections*, *Phys. Rev.* **D 63** (2001) 025008 [hep-ph/0006266];
D.G. Cerdeño, E. Gabrielli, S. Khalil, C. Muñoz and E. Torrente-Lujan, *Determination of the string scale in D-brane scenarios and dark matter implications*, *Nucl. Phys.* **B 603** (2001) 231 [hep-ph/0102270].